CORRESPONDENCE.

ON CERTAIN FORMULÆ IN MR. DAVID JONES'S WORK ON ANNUITIES.

To the Editor.

Sir,—I am not aware whether what seems to me to be a serious error in one of the expressions given in Jones's work *On Annuities and Reversionary Payments*, for a deferred assurance, has ever been noticed or not.

On page 170, vol. i., of the work referred to, the general value of a deferred assurance on any number of lives is given as

$$A_{\overline{(m,\, m_1,\, m_2,\, \&c.)}\rceil t}^{\,v} = r^{t+1} p_{\overline{(m,\, m_1,\, m_2,\, \&c.)},t}^{\,v} - (1-r) a_{\overline{(m,\, m_1,\, m_2,\, \&c.)}\rceil t}^{\,v}. \qquad (1)$$

An expression correct enough: but wishing, I presume, to give this result in a more convenient form, the author goes on to manipulate the latter term of the second member of this equation, thus—

$$(1-r) a_{\overline{(m,\, m_1,\, m_2,\, \&c.)}\rceil t}^{\,v} = (1-r) r^t p_{\overline{(m,\, m_1,\, m_2,\, \&c.)},t}^{\,v} \,.\, a_{\overline{(m+t,\, m_1+t,\, m_2+t,\, \&c.)}}^{\,v};$$

and in this way he deduces

$$A_{\overline{(m,\, m_1,\, m_2,\, \&c.)}\rceil t}^{\,v} = r^t p_{\overline{(m,\, m_1,\, m_2,\, \&c.)},t}^{\,v} \,.\, A_{\overline{(m+t,\, m_1+t,\, m_2+t,\, \&c.)}}^{\,v}. \qquad (2)$$

This latter expression it seems to me is quite erroneous, in consequence of a fatal error in the process that leads to it. In fact, $(1-r) r^t p_{\overline{(m,\, m_1,\, m_2,\, \&c.)},t}^{\,v}$ $.\, a_{\overline{(m+t,m_1+t,m_2+t,\&c.)}}^{\,v}$ is not $\left(\text{except in the single case when } v = m + m_1 + m_2 + \&c. \atop = \textit{number} \text{ of lives}\right)$ the equivalent of $(1-r) a_{\overline{(m,\, m_1,\, m_2,\, \&c.)}\rceil t}^{\,v}$, as assumed in the work referred to.

This will at once be seen if we particularise the expression. Suppose two lives (m and m_1) concerned, and $v=1$; then

$$(1-r) a_{\overline{(m,m_1)}\rceil t}^{\,1} = (1-r)(a_{m\rceil t} + a_{m_1\rceil t} - a_{m,m_1\rceil t})$$

$$= r^t (1-r)\left(\frac{l_{m+t}}{l_m} a_{m+t} + \frac{l_{m_1+t}}{l_{m_1}} a_{m_1+t} - \frac{l_{m+t} l_{m_1+t}}{l_m l_{m_1}} a_{m+t,m_1+t}\right),$$

while

$$(1-r) r^t p_{\overline{(m,m_1)},t}^{\,1}\, a_{\overline{m+t,m_1+t}}^{\,1} = r^t (1-r)\left(\frac{l_{m+t}}{l_m} + \frac{l_{m_1+t}}{l_{m_1}} - \frac{l_{m+t} l_{m_1+t}}{l_m l_{m_1}}\right)(a_{m+t} + a_{m_1+t} - a_{m+t,m_1+t});$$

a very different expression indeed, and giving rise to a very serious error.

As an illustration, let us take an actual case:—Required the single premium for £1 payable on death of last of two parties now aged 30 and 40 respectively, provided that event happens after 10 years (Carlisle 3 per cent.)

The true value, *i.e.*, the result of equation (1) is ·3118
while the value given by equation (2) is ·2959

a result too little by nearly 2 per cent.

Hoping you will be good enough to inform me whether this point has ever been remarked on previously,

I am, Sir, your most obedient servant,

JAMES R. MACFADYEN.

City of Glasgow Life Assurance Company,
Glasgow, 18th July, 1866.